647510 **BSCgraCC6010** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination

Oct/Nov -2020

GRAPH THEORY AND COMPLEX ANALYSIS -2 (CORE)

Time: 2:00 Hours **Marks: 56**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any four of the following questions.

Q-1

(A) ATTEMPT ANY TWO: **10**

(1) Prove that a simple graph with $\boldsymbol{n\ vertices\ and\ k\ components}$ can have atmost $\frac{1}{2}(\boldsymbol{n-k})(\boldsymbol{n-k+1})$ number of edges.

(2) Define: Euler Graph

Prove that a connected graph G is an Euler graph ***if and only if*** all vertices of G are of even degree.

(3) Define: Spanning Tree

Prove that a connected graph G with $\boldsymbol{n\ vertices}$ & $e\ edges$ is having exactly $\boldsymbol{n-1}$ tree branches and $\boldsymbol{e-n+1}$ chords with respect to any spanning tree.

(B) ATTEMPT ANY ONE : **04**

(1) What will be the smallest integer $\boldsymbol{n}$ for which the complete graph $\boldsymbol{K_n}$ has atleast 123 edges?

(2) Explain graph operations **Deletion** and **Fusion** by taking suitable graphs.

Q-2

(A) ATTEMPT ANY TWO: **10**

(1) Define: Planar Graph

Prove that $\boldsymbol{K_{3,3}}$, the regular graph with 6 vertices and 9 edges is non-planar.

(2) State and prove the Euler's formula for connected planar graphs.

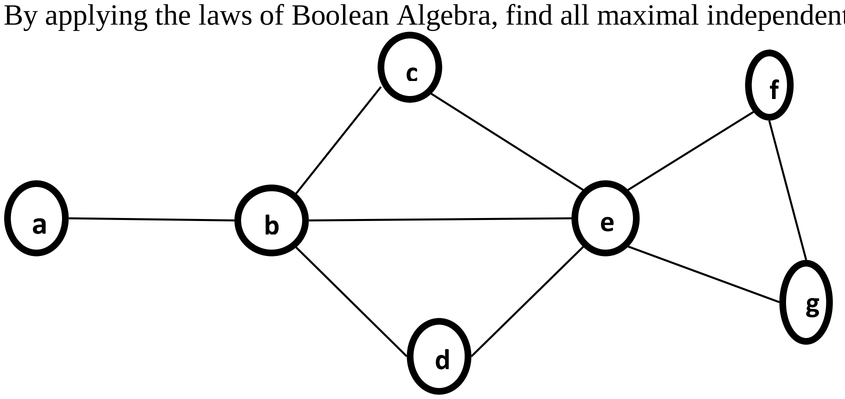
(3) Define: Incidence matrix.

Prove that the rank of the incidence matrix $\boldsymbol{A(G)}$ of a connected graph $\boldsymbol{G}$ with $\boldsymbol{n}$ vertices is $\boldsymbol{n-1}$.

(B) ATTEMPT ANY ONE : **04**

(1) Write a short note on Geometric Dual of a graph.

(2) By applying the laws of Boolean Algebra, find all maximal independent sets of the following graph.



Q-3

(A) ATTEMPT ANY TWO: **10**

(1) Define: Singular bilinear mapping.

Let $\boldsymbol{w_1, w_2, w_3}$ be the images of $\boldsymbol{z_1, z_2, z_3}$ respectively of $\boldsymbol{Z}$ −plane in $\boldsymbol{W}$ −plane under a bilinear transformation

$\boldsymbol{w = f(z) = \frac{az+b}{cz+d}, where\ ad - bc \neq 0}$, then obtain the singular bilinear mapping.

(2) Prove that the Mobious Transformation maps circles or straight lines into circles or straight lines.

(3) Prove that the mapping $\boldsymbol{w = z^2}$ is a Conformal Mapping.

(B) ATTEMPT ANY ONE : **04**

(1) Discuss the transformation $w = \cos z$.

(2) Prove that the transformation $w = z^2 + 2z$ maps the unit circle $|z| = 1$ of the Z −plane into a cardioid in the W −plane.

Q-4

(A) ATTEMPT ANY TWO: **10**

(1) State and prove the Taylor's infinite series of an analytic function $f(z)$.

(2) State and prove Laurent's infinite series of an analytic function $f(z)$.

(3) Let $a \in (-1, 1)$ then derive Laurent's series representation

$\frac{a}{z-a} = \sum_{n=1}^{\infty} \frac{a^n}{z^n}$, $(|a| < |z| < \infty)$ and deduce the followings from it.

$$\sum_{n=1}^{\infty} a^n \cos n\theta = \frac{a\cos\theta - a^2}{1 - 2a\cos\theta + a^2} \quad \underline{\text{and}} \quad \sum_{n=1}^{\infty} a^n \sin n\theta = \frac{a\sin\theta}{1 - 2a\cos\theta + a^2}$$

(B) ATTEMPT ANY ONE : **04**

(1) Prove that $\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \frac{z^2}{4!} + \cdots$, $when\ z \neq 0$.

(2) Find the Maclaurin series expansion of $f(z) = \frac{z}{z^4+9}$.

Q-5

(A) ATTEMPT ANY TWO: **10**

(1) Define: (i) Simple Pole (ii) Pole of order m

If z_0 is the pole of order $m (m \geq 2)$ of a complex function $f(z)$, then prove that

$Res(f(z), z_0) = \frac{\varphi^{m-1}(z_0)}{(m-1)!}$, $where\ \varphi(z) = (z - z_0)^m f(z)$.

(2) State and prove the Cauchy's residue theorem.

(3) Evaluate: $\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$, $C: |z - 2| = 2$.

(B) ATTEMPT ANY ONE: **04**

(1) Evaluate: $\int_0^{\infty} \frac{\cos ax}{x^2+1} dx, a > 0$.

(2) Evaluate: $\int_0^{2\pi} \frac{d\theta}{(5+4\sin\theta)}$

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